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Technical Notes

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Efficiency Analysis of a Two-Stage Hybrid Electric Thruster

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Nomenclature

c_p = specific heat
I = current
$\dot{m}$ = mass flow rate
P = power
T = temperature
U = voltage
η = efficiency

Subscripts

cal = calorimeter
dc = direct current (first stage)
el = electric
H_2O = cooling water
in = inflow, coolant
ind = induced
inj = injection head
kin = kinetic
ng = neutral gear
out = outflow, coolant
Pl = plasma
RF = radio frequency, second stage
th = thermal (loss)
tot = total (both stages)
tube = discharge tube

I. Introduction

ARCJET thrusters have been developed throughout the past few decades. They provide relatively high thrust at moderate exhaust velocities, and their plasma flows are characterized by high specific enthalpy and high flow velocity. These characteristics are, however, combined with the presence of steep radial property gradients as in a hot, energy-rich core with a relatively cold gas layer at its edge.

When the limits to the nozzle exit velocity are imposed by wall and electrode materials, another possibility to transfer more power into an arc heated plasma flow is to reheat the relatively cold plume edges by means of another heating mechanism, as done in an afterburner. TIHTUS has been developed over the past four years and is a novel two-stage plasma thruster where reheating of the arc heated plume is realized by means of inductive heating, as sketched in Fig. 1 [1–3]. It is therefore considered the predecessor of a future propulsion system for the transport of heavy payload on interplanetary trajectories [1]. The principal question within the development of this thruster is whether it is possible to specifically heat the outer edges of an arcjet plume so that higher exhaust velocity can be attained. A dependency of the power staging between the two thruster stages and of the gas mass flow rate staging is expected.

For the presented research, the thruster was investigated with a cavity calorimeter which provides data of plasma power so that a total efficiency can be derived. Moreover, a method for deducing the two-stage system's efficiency from operational data is presented and its data are compared with the measured calorimetric data.

II. Experimental Apparatus

The two-stage plasma thruster is currently under investigation using electric propulsion diagnostics. The present paper presents data of calorimetric measurement. The thruster was also investigated by a baffle plate, a gas dynamic pitot probe, and electric time-of-flight probes to evaluate the thruster's output kinetic energy [2,3]. These measurements are, however, not part of the present paper.

For investigating purposes, it is installed in a ground test facility at Institute of Space Systems (IRS, Insitut für Raumfahrtsysteme) consisting of the two plasma thruster stages and a vacuum chamber. The size of the chamber is 3 m in length and 2 m in diameter. Installed are a gas supply system, water-cooling system, and a data acquisition system. The lid of the vacuum chamber carries the thruster. The rear end of the chamber is connected to a vacuum pump system, the total suction power of which amounts to 6000 m^3/h at atmospheric pressure or 250,000 m^3/h at 10 Pa. The two-stage plasma thruster TIHTUS consists of an arcjet thruster (first stage) and an inductively heating afterburner (second stage). The thruster is supplied by a 6 MW dc power supply and a 180 kW radio-frequency power supply, respectively. The thruster is currently operated as a water-cooled model using hydrogen as propellant. Therefore, thermal powers such as tube cooling power loss or resonant circuit power are surveyed using resistance thermometers. However, at a further stage of development, the strategy foresees building the plasma source in a radiation-cooled design, promising an additional gain in specific impulse.

The first stage is formed by the arcjet thruster HIPARC-W [4]. It has a power draw of up to 100 kW and a converging–diverging nozzle with a throat diameter of 6 mm and an exit diameter of 65 mm. The second stage consists of a cylindric 270-mm-long discharge tube and a coil of a diameter of 100 mm spun around it as part of a resonance circuit. With 3.5 coil windings and four capacitors connected, the nominal operational frequency results in 840 kHz. In induction heating, the power is coupled into the plasma at a near-coil position due to the skin effect [5]. In the arcjet plume, this is where the relatively cold gas layer is located. The alternating RF current in the

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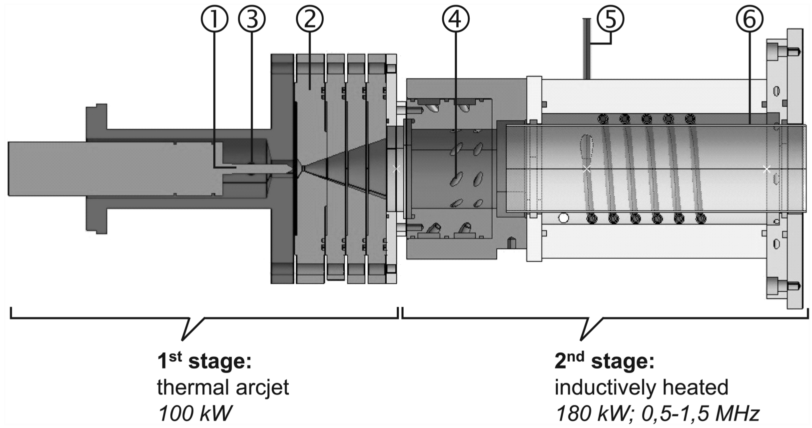

Fig. 1 Setup of TIHTUS; 1: cathode, 2: anode, 3: central gas supply, 4: swirl gas supply, 5: induction coil, 6: discharge tube.

coil induces an oscillating magnetic field inside the tube. This field initiates an electric discharge in the propellant oriented in the opposite direction of the coil current. The free electrons contained in the plasma from the thermal arcjet are accelerated by the electric field and, by means of collisions, they transfer their induced power to the atoms and molecules.

In this novel thruster, power may be coupled into either the arc heated (dc) or the inductively heated (RF) stage or both. Central gas flow through the arc heated stage is expanded first into the injection head of the inductive, second stage where a swirl gas flow can be admixed. Each operational condition of TIHTUS is therefore referred to as T $P_{\mathrm{dc}}|P_{\mathrm{RF}} - \dot{m}_{\mathrm{dc}}|\dot{m}_{\mathrm{RF}}$ throughout this paper. As an example, T 25|25–200|100 refers to the operating condition of TIHTUS in which 25 kW are coupled into both first and second stage, while a gas flow rate of 200 mg/s is supplied to the arcjet stage and one of 100 mg/s is supplied to the inductively heated thruster stage. During operation, the voltage U_{dc} and current I_{dc} of the arcjet stage and U_{RF} and I_{RF} of the inductively coupled stage are measured. The latter are plate voltage and current measured at the triode of the resonant circuit. The power spent on the respective stage is thus $P_{\mathrm{el}} = UI$. It is stated that the plasma power

$$P_{\mathrm{Pl,dc}} = P_{\mathrm{dc}} - P_{\mathrm{th,dc}} \tag{1}$$

is the power available in the plasma from the arcjet stage. The thermal power loss to the coolant P_{th} is calculated from the measured increase of the temperature ΔT of the cooling water at mass flow rate $\dot{m}_{\mathrm{H_2O}}$ right before and after the thruster stage, according to

$$P_{\mathrm{th}} = \dot{m}_{\mathrm{H_2O}} c_{p,\mathrm{H_2O}} \Delta T \tag{2}$$

In the case of the inductive stage, two different kinds of thermal loss $P_{\mathrm{th,RF}} = P_{\mathrm{th,tube}} + P_{\mathrm{th,inj}}$ are observed: the thermal loss to the cooling water through the discharge tube and through the injection head. They are measured in the same fashion.

However, plasma power is also measured directly. For this purpose, a cone-shaped, copper cavity calorimeter was developed to determine thermal plasma power, and is shown in Fig. 2 [6]. The plasma ejected from the plasma thruster enters the cavity through the entrance aperture which has a diameter of 120 mm, which is approximately 25% larger than the plasma beam. The distance between the calorimeter and the plasma outlet of TIHTUS is 70 mm and large enough so that the discharge behavior of the plasma thruster is not disturbed. The copper walls are heated by radiation, convection, and wall-recombination. The entire cavity is equipped with spiral copper tubes on the outside that guide the cooling water, as can be seen in Fig. 2. The water cools the copper wall so that the plasma is completely cooled and the entire power of the plasma is transferred to the cooling water. The plasma power is then determined analogous to Eq. (2). Because the calorimeter is set up only 70 mm downstream of the thruster exit, flow entrainment of gas from the vacuum chamber is neglected.

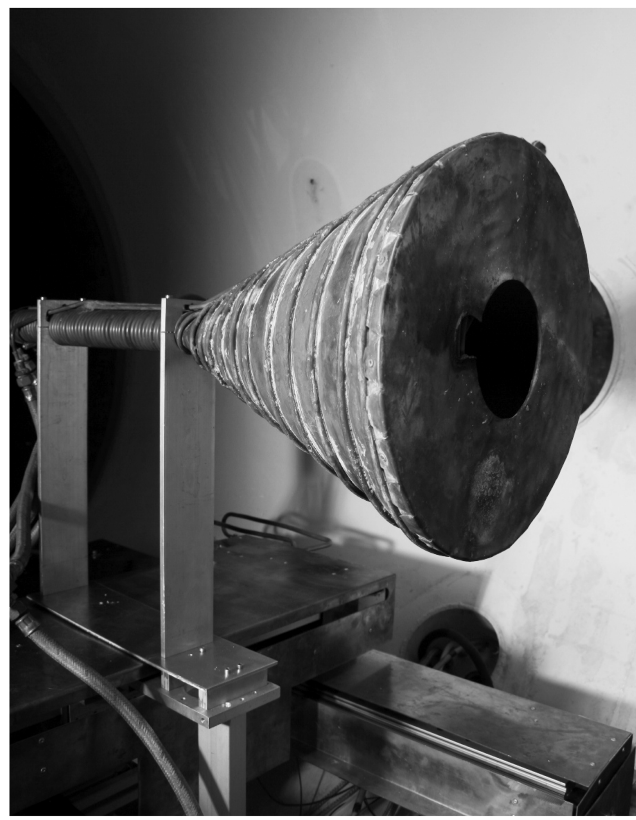

Fig. 2 Cavity calorimeter.

III. Results with Hydrogen

The present investigation presents thermal efficiencies of the two-stage thruster where the power in the fluid is related to input power. The results of the current investigations show how plasma power can be estimated from operation supervisory data and differ from directly measured data below 10%. This makes direct measurement of the plasma power by a calorimeter or other techniques necessary only when exact knowledge of the power present in the plasma is required. It will also be shown how, from the efficiencies of the respective stages, an overall efficiency can be derived for the entire device as one thruster.

For the second (inductive) stage, apart from thermal losses, another loss was reported by Herdrich et al. which is referred to as the neutral gear power [6]. Neutral gear power is lost to the components of the facility, e.g., in the form of electromagnetic emission or to the

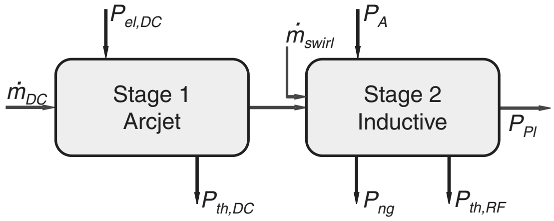

Fig. 3 Staging principle.

oscillator tube of the RF generator, amongst others. The neutral gear power P_{ng} can be assessed during operation without a plasma load [6]. Thus, only a part of the anode power P_{RF} is induced into the plasma as $P_{\mathrm{ind}} = P_{\mathrm{RF}} - P_{\mathrm{ng}}$. Again, from the power induced into the plasma, a part is lost to the cooling water, indicated by $P_{\mathrm{th,RF}}$ and measured as previously described

$$P_{\mathrm{Pl,RF}} = P_{\mathrm{RF}} - P_{\mathrm{ng}} - P_{\mathrm{th,RF}} \tag{3}$$

Figure 3 shows a principle of the two-stage thruster TIHTUS. Sketched are the gas flow, the input power to each stage, and the power losses. It is understood from Fig. 3 that in the first stage, although as part of a two-stage system, the plasma power remains the same as in Eq. (1). Meanwhile, the second stage is supplied with two kinds of power: the plasma power generated by the first stage of TIHTUS and the anode power, so that the power of the plasma ejected from the second stage ($P_{\mathrm{Pl,RF}} = P_{\mathrm{Pl}}$) yields

$$P_{\mathrm{Pl}} = P_{\mathrm{Pl,dc}} + P_{\mathrm{RF}} - P_{\mathrm{ng}} - P_{\mathrm{th,RF}} \tag{4}$$

Substituting for $P_{\mathrm{Pl,dc}}$ results in

$$P_{\mathrm{Pl}} = P_{\mathrm{dc}} + P_{\mathrm{RF}} - P_{\mathrm{ng}} - P_{\mathrm{th,dc}} - P_{\mathrm{th,RF}} \tag{5}$$

In section A of Table 1, the particular conditions investigated in the present research are presented according to the nomenclature explained previously. The power input to the stages sums up to 50 kW for each condition, whereas the distribution to the stages varies. Mass flow rate to the system adds up to 300 mg/s, again at varying distribution to the two stages of the hybrid thruster. To prove Eq. (5) true, Table 1 also compares its results to the plasma power measured with the calorimeter described previously. The table thus shows the results and that, for hydrogen operation, the method of determining plasma power from operational data differs from direct measurement less than 10%.

Table 1 Plasma power and thermal efficiency: directly measured and determined from operational data

A)	$P_{\mathrm{Pl,cal}}$, kW	P_{Pl}, acc. to Eq. (5), kW	Difference, %
T 50|00–300|0	25.56	25.56	0
T 25|25–300|0	20.39	22.16	8.7
T 20|30–300|0	21.16	21.73	2.7
T 25|25–200|100	21.08	19.99	5.2
T 25|25–100|200	21.32	19.54	8.4
B)	$\eta_{\mathrm{th,cal}}$, %	η_{th}, acc. to Eq. (11), %	Difference, %
T 50|00–300|0	51.04	51.04	0
T 25|25–300|0	40.67	43.80	1.6
T 20|30–300|0	41.73	42.41	7.6
T 25|25–200|100	42.87	42.00	2.0
T 25|25–100|200	42.31	37.80	10.7
C)	$\eta_{\mathrm{th,dc}}$, %	$\eta_{\mathrm{th,RF}}$, %	——
T 50|00–300|0	78.72	64.84	——
T 25|25–300|0	76.98	49.46	——
T 20|30–300|0	78.18	46.50	——
T 25|25–200|100	77.38	47.40	——
T 25|25–100|200	60.61	46.98	——

The general definition of efficiency is

$$\eta = \frac{\text{usable power}}{\text{supplied power}} \tag{6}$$

Thus, in the present case, thermal efficiency is, according to Fig. 3, defined as

$$\eta_{\mathrm{th,tot}} = \frac{P_{\mathrm{Pl}}}{P_{\mathrm{dc}} + P_{\mathrm{RF}} + P_{\mathrm{kin}}} \tag{7}$$

However, it is assumed that the input kinetic power from the gas injection system of the first and second stages is small compared with the high electric power, and is therefore not taken into account. The power in the plasma P_{Pl} exhausted from TIHTUS is therefore

$$P_{\mathrm{Pl}} = \eta_{\mathrm{th,tot}}(P_{\mathrm{dc}} + P_{\mathrm{RF}}) \tag{8}$$

but with Eqs. (4) and (6) and Fig. 3, it can also be expressed as

$$P_{\mathrm{Pl}} = \eta_{\mathrm{th,RF}}(P_{\mathrm{Pl,dc}} + P_{\mathrm{RF}}) \tag{9}$$

where $P_{\mathrm{Pl,dc}} = \eta_{\mathrm{th,dc}} P_{\mathrm{dc}}$, so that

$$P_{\mathrm{Pl}} = \eta_{\mathrm{th,RF}}(P_{\mathrm{dc}}\eta_{\mathrm{th,dc}} + P_{\mathrm{RF}}) \tag{10}$$

Equating Eqs. (8) and (10) yields

$$\eta_{\mathrm{th,tot}} = \eta_{\mathrm{th,RF}}\left(1 + \frac{P_{\mathrm{dc}}}{P_{\mathrm{dc}} + P_{\mathrm{RF}}}(\eta_{\mathrm{th,dc}} - 1)\right) \tag{11}$$

This consideration is proven correct by the measurements presented in section B of Table 1. In section C of Table 1, the thermal efficiencies of each stage are disclosed. The table shows that this consideration differs as little as 10.7% from directly assessed data. This shows that Eq. (11) is a good approximation for the thermal efficiency of the thruster. With it, thermal efficiency of the two-stage thruster can be monitored by operational data. The uncertainty of the calorimetric measurement data is determined by $\Delta P_{\mathrm{el}} = U\Delta I + I\Delta U$, and $\Delta P_{\mathrm{th}} = c_{p,\mathrm{H_2O}}\dot{m}_{\mathrm{H_2O}}(\Delta T_{\mathrm{out}} + \Delta T_{\mathrm{in}}) + c_{p,\mathrm{H_2O}}\Delta\dot{m}_{\mathrm{H_2O}}(T_{\mathrm{out}} - T_{\mathrm{in}})$, as well as $\Delta\eta_{\mathrm{th}} = \Delta P_{\mathrm{el}} P_{\mathrm{th}}/P_{\mathrm{el}}^2 + \Delta P_{\mathrm{th}}/P_{\mathrm{el}}$ and yields 9%. It is assumed that the difference in the plasma power measurements originates from the measurement uncertainties and uncertainties in the determination of neutral gear power.

The total thermal efficiency of the two-stage device is, as seen from Eq. (11), dependent on the efficiencies of each stage $\eta_{\mathrm{th,RF}}$ and $\eta_{\mathrm{th,dc}} = f(P_{\mathrm{dc}}, \dot{m}_{\mathrm{dc}})$, and on how the power is distributed to the two stages indicated by the factor $P_{\mathrm{dc}}/(P_{\mathrm{dc}} + P_{\mathrm{RF}})$. For example, the ratio of powers is $P_{\mathrm{dc}}/(P_{\mathrm{dc}} + P_{\mathrm{RF}}) = 0$ when the entire power is supplied to the second stage. The total efficiency of the system, then, equals $\eta_{\mathrm{th,tot}} = \eta_{\mathrm{th,RF}}$. It is known that the thermal efficiency of inductively heated devices is quite low compared with high thermal efficiencies of 80–90% of thermal arcjet thrusters. At the operating condition T 0|30–300|, where 30 kW are supplied to the second stage without the arcjet stage in operation, the thermal efficiency $\eta_{\mathrm{th,RF}} = 41.8\%$. If the ratio of powers, on the contrary, is $P_{\mathrm{dc}}/(P_{\mathrm{dc}} + P_{\mathrm{RF}}) = 1$, the total invested power is supplied to the first stage. In this case, the plasma sequentially flows first through the operating arc heated stage and then through the inductively heated stage, without being heated there. Equation (11), in that case, yields a multiplication of the single efficiencies for the total efficiency as in $\eta_{\mathrm{th,tot}} = \eta_{\mathrm{th,RF}}\eta_{\mathrm{th,dc}}$. It is important to note that the coupling efficiency $\eta_{\mathrm{th,RF}}$ of the inductively heated stage is dependent on the state of the plasma influenced by both the first- and second-stage operating condition. The coupling efficiency is strongly dependent on the electric conductivity of the plasma, where the power and mass flow rate of the first stage play a role, but also power and mass flow rate of the second stage, as well as injection angle. Hence, $\eta_{\mathrm{th,RF}} = f(P_{\mathrm{dc}}, \dot{m}_{\mathrm{dc}}, P_{\mathrm{RF}}, \dot{m}_{\mathrm{RF}})$, indicating that $\eta_{\mathrm{th,RF}}$ with the first stage active can overtop $\eta_{th,RF}$ with the first stage inactive. The total efficiency $\eta_{th,tot}$, therefore, varies as in $\eta_{th,RF} < \eta_{th,tot} < \eta_{th,dc}\eta_{th,RF}$. From the measurement data in Table 1, it can be seen how for hybrid operating conditions at a total of 50 kW, the thermal efficiency is lower than at pure arc heated condition, as in T 50|00–300|. A major effort must be made to improve the inductive stage's thermal efficiency.

IV. Conclusions

A novel two-stage, electric thruster TIHTUS is under development at IRS. The first stage is an arcjet, whereas the second stage is inductive heating of the arcjet plume. Because of the unknown power losses to the facility components in the inductively heated stage, a cavity calorimeter is used to determine plasma power. It is measured over a range of power and mass flow ratios between the two stages. An analytical approach is used, in which plasma power can be approximated from operational data differing only by 10% from the plasma power measured by the calorimeter. Using a novel approach of determining efficiency from the efficiencies of the respective single stages, the thermal efficiency of the complex two-stage thruster is determined and compared with measurement data. The agreement is good with 10.7%.

For this consideration, the efficiency of the first stage is given as $\eta_{th,dc} = f(P_{dc}, \dot{m}_{dc})$ and the efficiency of the second stage is given as $\eta_{th,RF} = f(P_{dc}, \dot{m}_{dc}, P_{RF}, \dot{m}_{RF})$.

As a next step, the dependency on the single stage efficiencies must be modeled, possibly from empirical data, to optimize the thruster. The optimum operating parameters for P_{dc}, $\dot{m}_{dc}$, $\dot{m}_{RF}$, and P_{RF} can therefore be found to increase efficiency.

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